CIS 580

# Machine Perception

Or *Geometric Computer Vision*

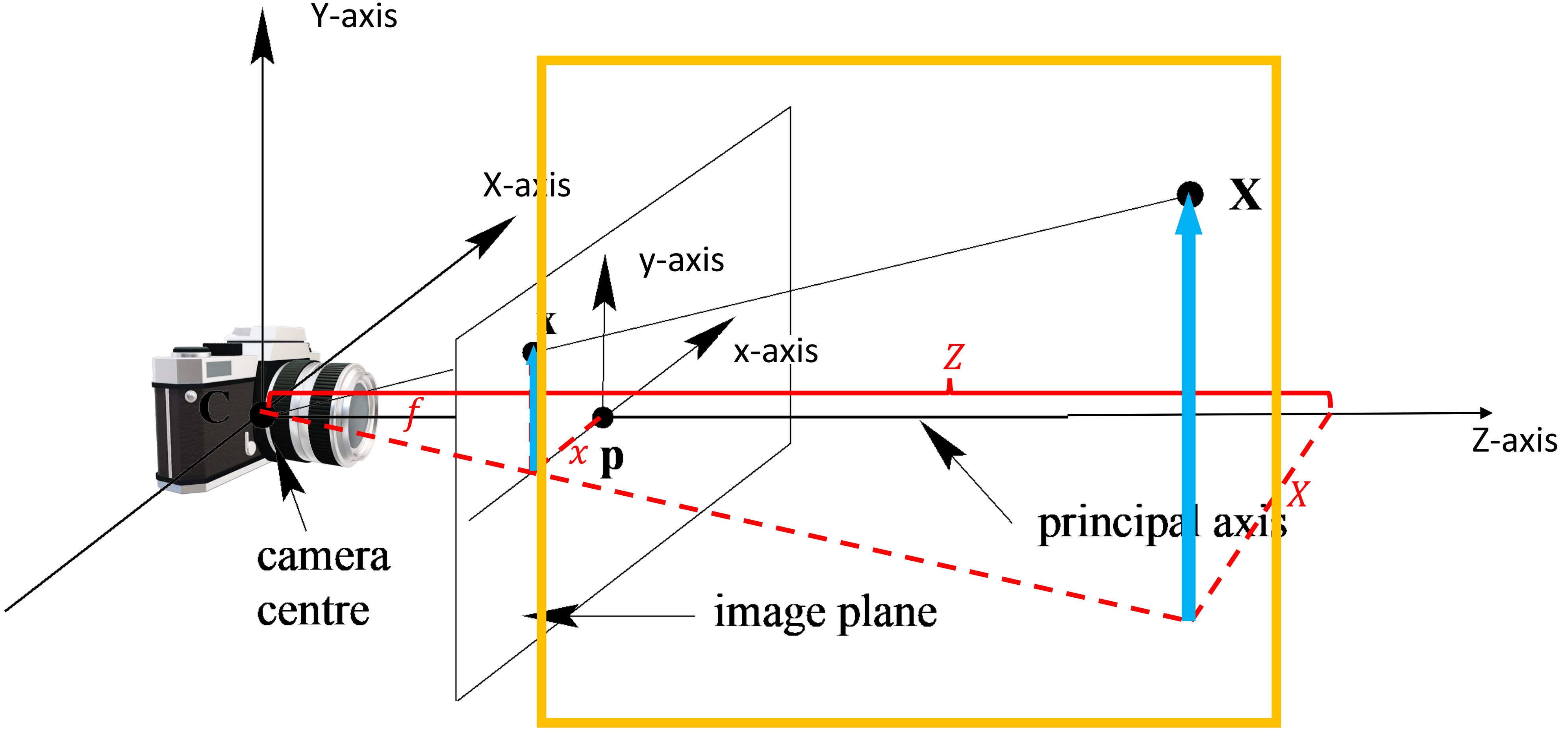
Instructor: Lingjie Liu

Lec 3: Jan 27, 2024

# We will start OH this week on Zoom

| | OH time | OH Zoom Link |
|---|---|---|
| Chuhao Chen | Mon 4:00-5:00pm | https://upenn.zoom.us/j/93800746935 |
| Xiangyu Han | Mon 9:30-11:30am | https://upenn.zoom.us/j/98703023093 |
| Aishwarya Balaji | Thur 1-2pm | https://upenn.zoom.us/j/91911855363?pwd=BfYEcbN8BmWxYQeocvejdevynvRBI1.1 |
| Yiming Huang | Mon 6pm - 8pm | https://upenn.zoom.us/j/8013153196 |
| Qiao Feng | Tue 10:30am-11:30am | https://upenn.zoom.us/j/97190861496 |
| Prakriti Prasad | Thursday 9-10 am | https://us04web.zoom.us/j/75830390501?pwd=5X2MOvaUwtazRHeis8Ma5SEHdhtZUN.1 |
| Paisley Hou | Mon 1:30-2:30 pm | https://zoom.us/j/92206750596?pwd=tOCsHrtDGJsOLhvz9aTqzk7Bue0aSb.1 |
| Pengyu Chen | Wed 4-6 PM | https://upenn.zoom.us/j/8224378933?omn=93556390130 |
| Xuyi Meng | Tue 9am-10am | https://upenn.zoom.us/j/9152271549 |
| Bryan Alfaro | Fri 3-4 PM | https://upenn.zoom.us/j/92114670861?pwd=XbTV6AAfwGZqle1JfV9tC6qZGR3Wn8.1 |
| Zi-Yan | Thur 10a-11p | https://us04web.zoom.us/j/71811908516?pwd=XbeesfxBVoap9QA21lywmkqJSebOOC.1 |
| Quan A. Pham | Tue 8:00 - 9:00 am | https://upenn.zoom.us/j/95713266263?pwd=WXVIu9CMnahpk1z25cClQbomoyS1b1.1 |
| Yicong Wang | Wed 8:15 - 9:15 am | https://upenn.zoom.us/j/98879627094?pwd=z3boRliz9dWw6Urf1aExWeoCp1DKtO.1 |
| Lingjie Liu | Fri 5-6pm | My office: Levine 462 |

# Recap: Basic Perspective Projection Equations

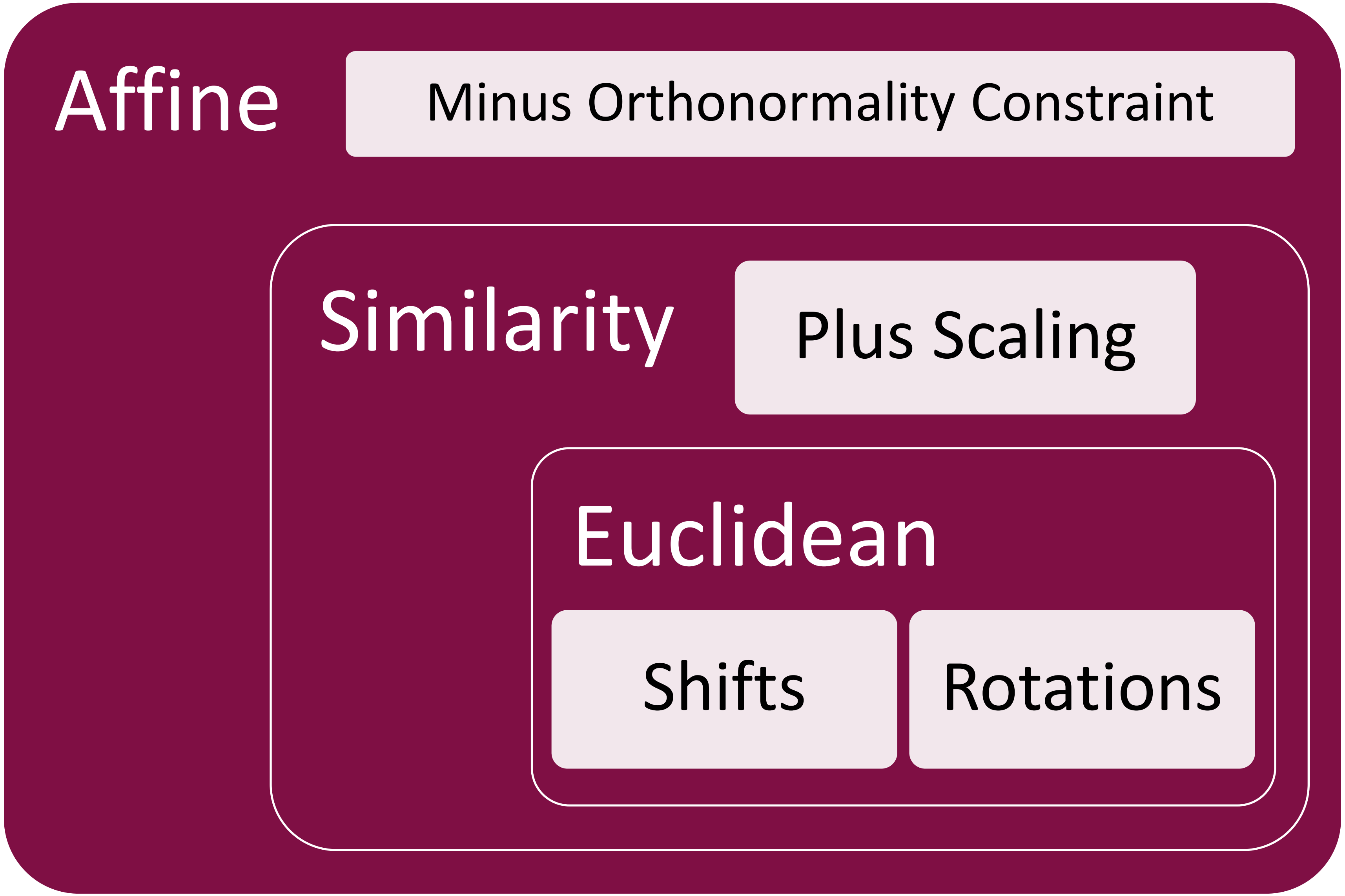



$$x = f\frac{X}{Z}, y = f\frac{Y}{Z}$$

# Recap: Points and Planes in Euclidean 3D (shortcut $\mathbb{R}^3$)

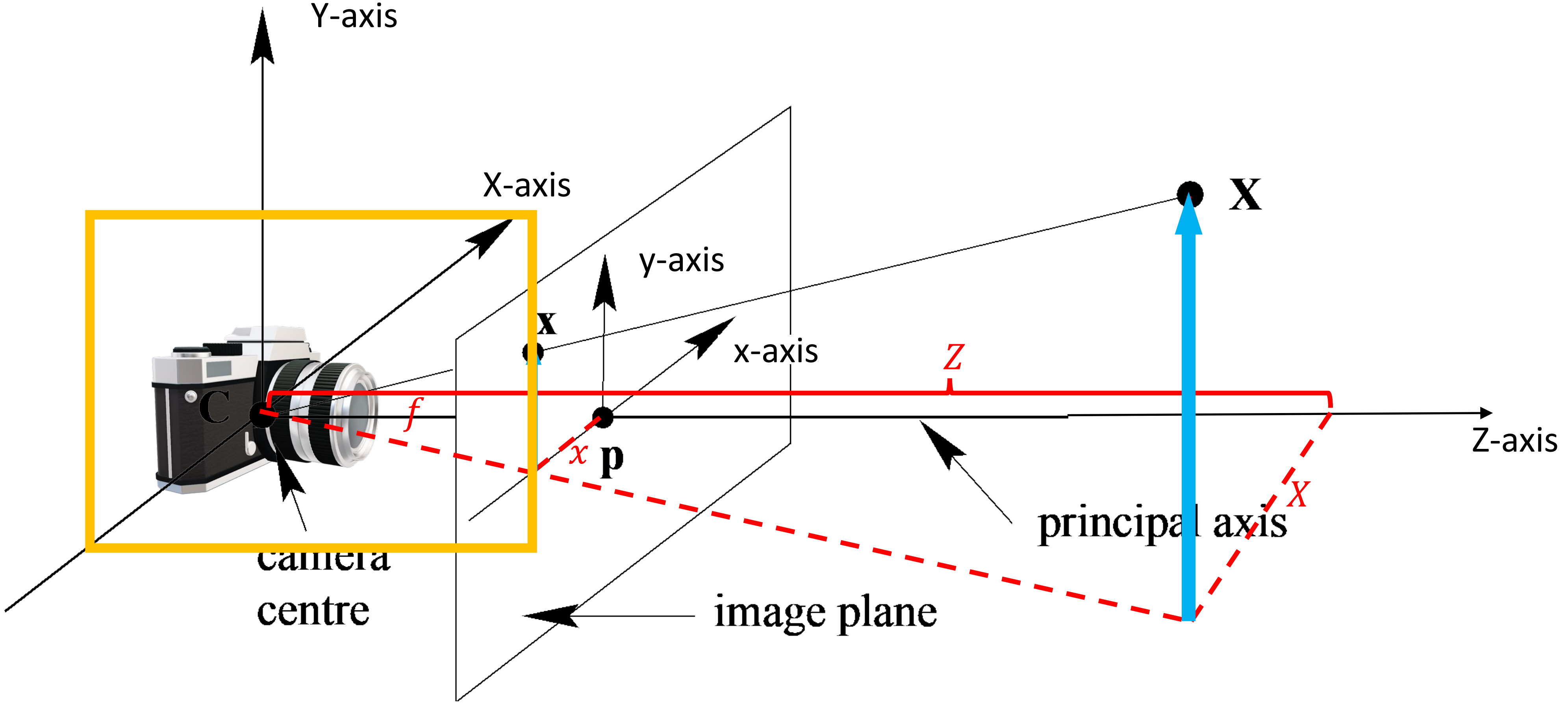
- Point in 3D space: $\boldsymbol{x} = (x, y, z)^T \in \mathbb{R}^3$
- Plane in 3D space:
  - $ax + by + cz + d = 0$
  - $\boldsymbol{\pi}^T \begin{pmatrix} x \\ y \\ z \\ {\color{red}1} \end{pmatrix} = 0, \text{where } \boldsymbol{\pi} = [a, b, c, d]^T$
- Line in 3D space:
  - $(x, y, z)^T = (x_1, y_1, z_1)^T + \lambda(x_2 - x_1, y_2 - y_1, z_2 - z_1)^T$ for some value of $\lambda$
  - $\begin{pmatrix} \boldsymbol{\pi}_1^T \\ \boldsymbol{\pi}_2^T \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ {\color{red}1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

# Recap: Hierarchy of Transformations



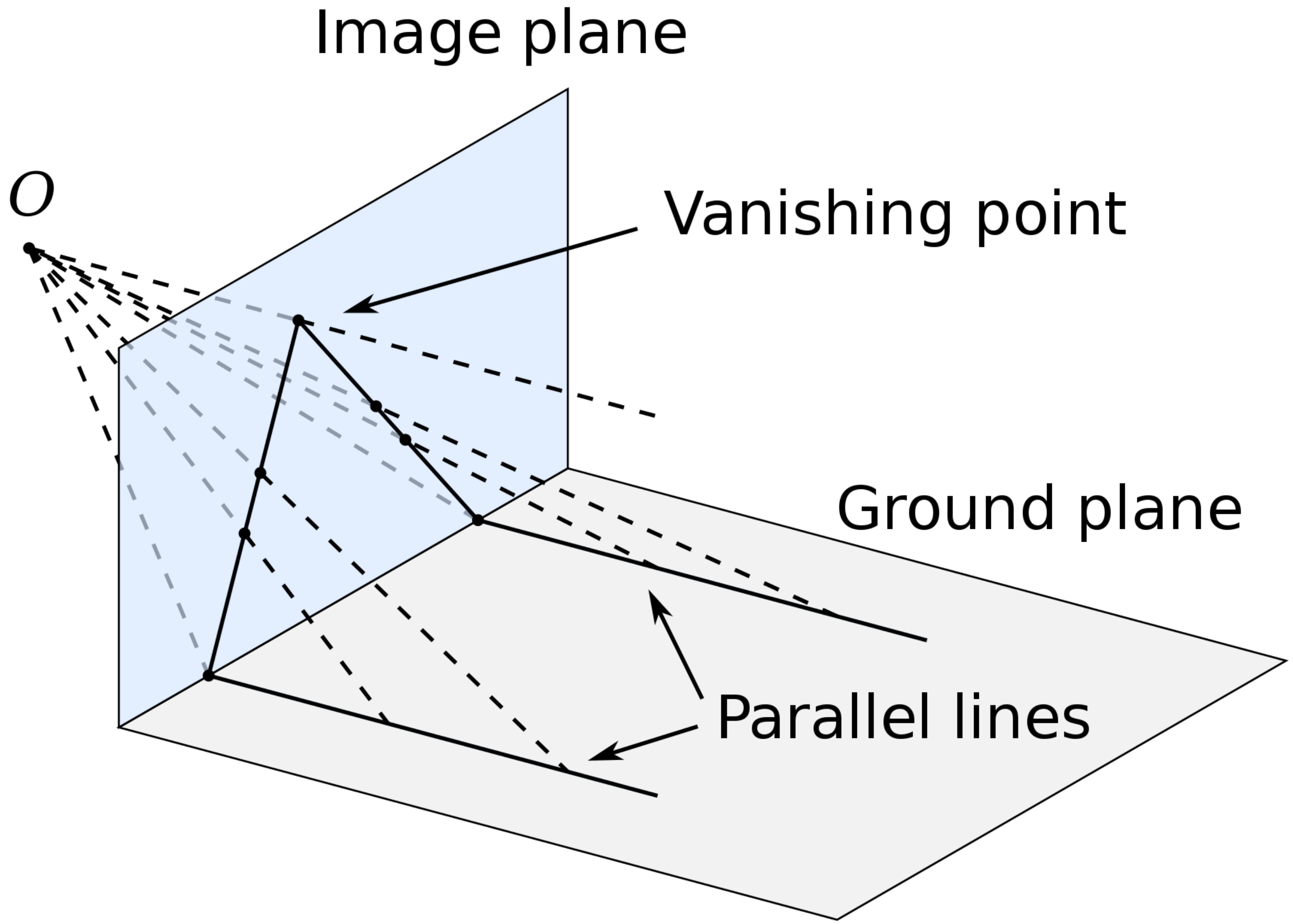

Soon an even more general transformation (“projective”)!

# Recap: Basic Perspective Projection Equations




$$x = f\frac{X}{Z}, y = f\frac{Y}{Z}$$

# Recap: Where vanishing points come from

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# Recap: Projective Geometry

- Extension of Euclidean geometry that deals with points at infinity
  - Other than that, Projective Geometry keeps many of the same features as Euclidean geometry.

- A key property is that two lines always meet in a point
  - (sometimes a point at infinity)

# Recap: "Homogeneous coordinates": Euclidean $\mathbb{R}^n \rightarrow$ Projective $\mathbb{P}^n$

***Homogeneous coordinates***

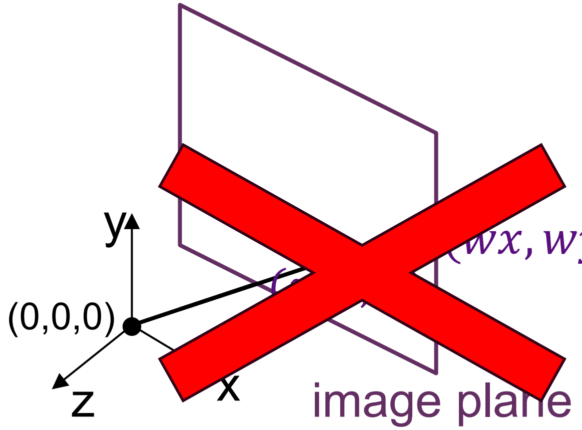
– represent coordinates in $\mathbb{P}^2$ with a 3-vector

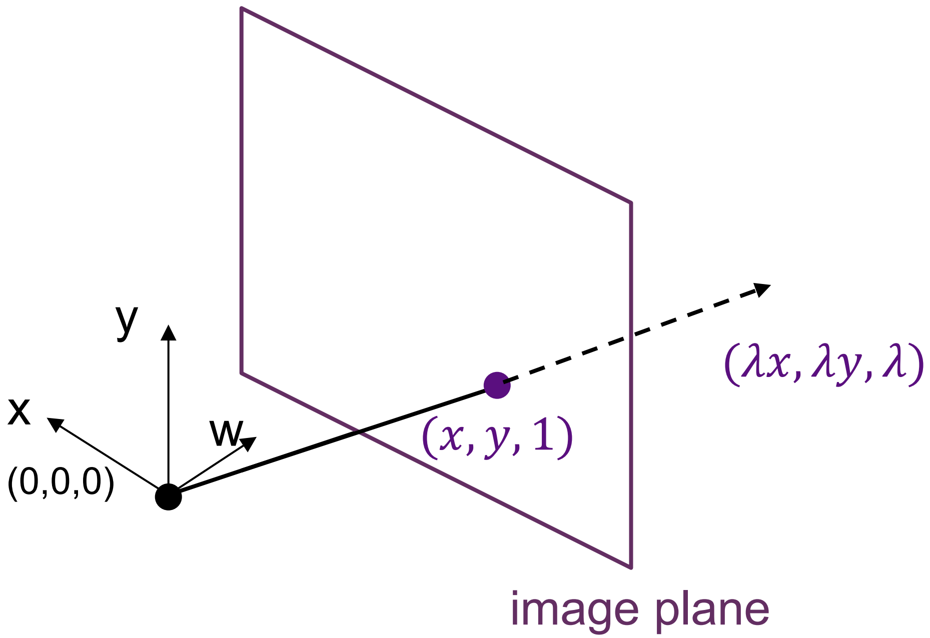
$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

We will normally use $w$ to denote this homogeneous vector element

# Recap: Projective $\mathbb{P}^n \rightarrow$ Euclidean $\mathbb{R}^n$

- $(x, y, w) = \left(\frac{x}{w}, \frac{y}{w}, 1\right), if\ x \neq 0$
- Geometric intuition



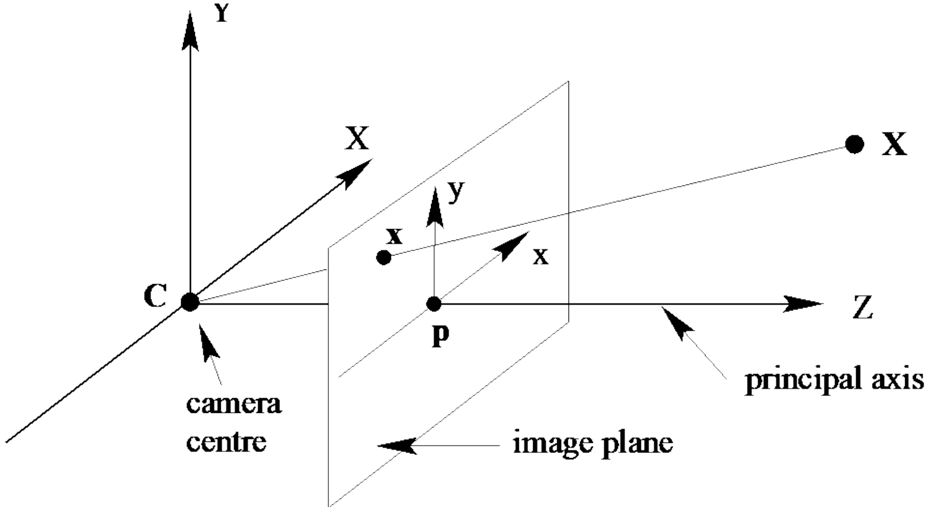




- $(x, y, 0)$: an infinity point
  Note: $\mathbb{P}^2$ does not contain $(0,0,0)$.

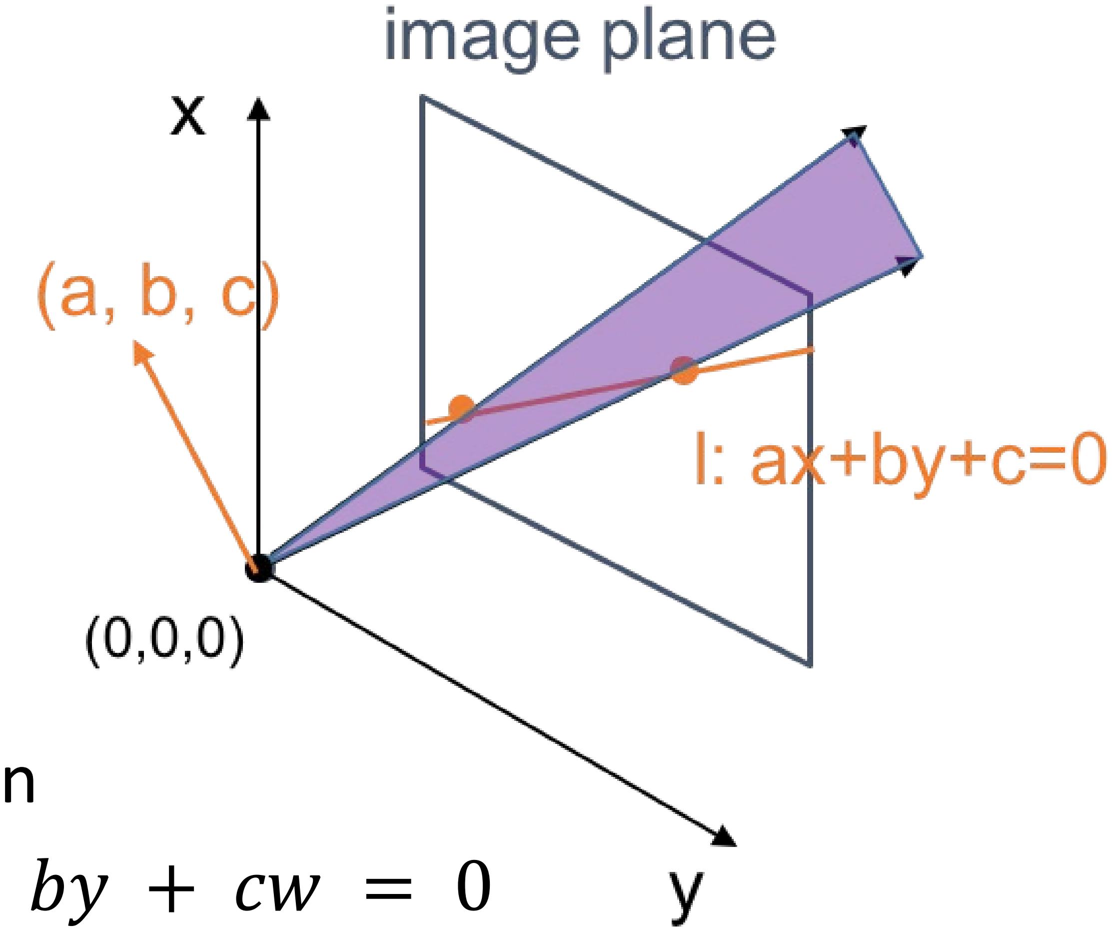
# Recap: Perspective Projections are Linear in $\mathbb{P}$

Recall: $\boxed{x = f\frac{X}{Z}, y = f\frac{Y}{Z}}$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}$$



# Recap: Projective lines




- A line is a *plane* of rays through origin
  - all rays $(x, y, w)$ satisfying: $ax + by + cw = 0$

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = 0$$

$$\boldsymbol{l}^T \boldsymbol{x} = 0$$

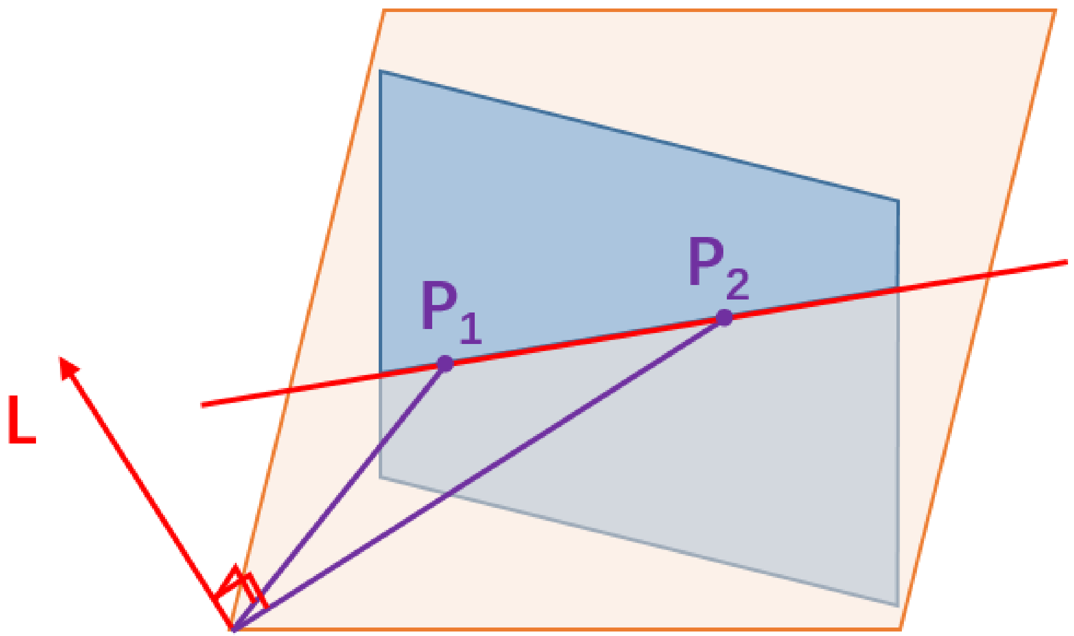
# Recap:Relationship btw Line and Point Representations in $\mathbb{P}^2$

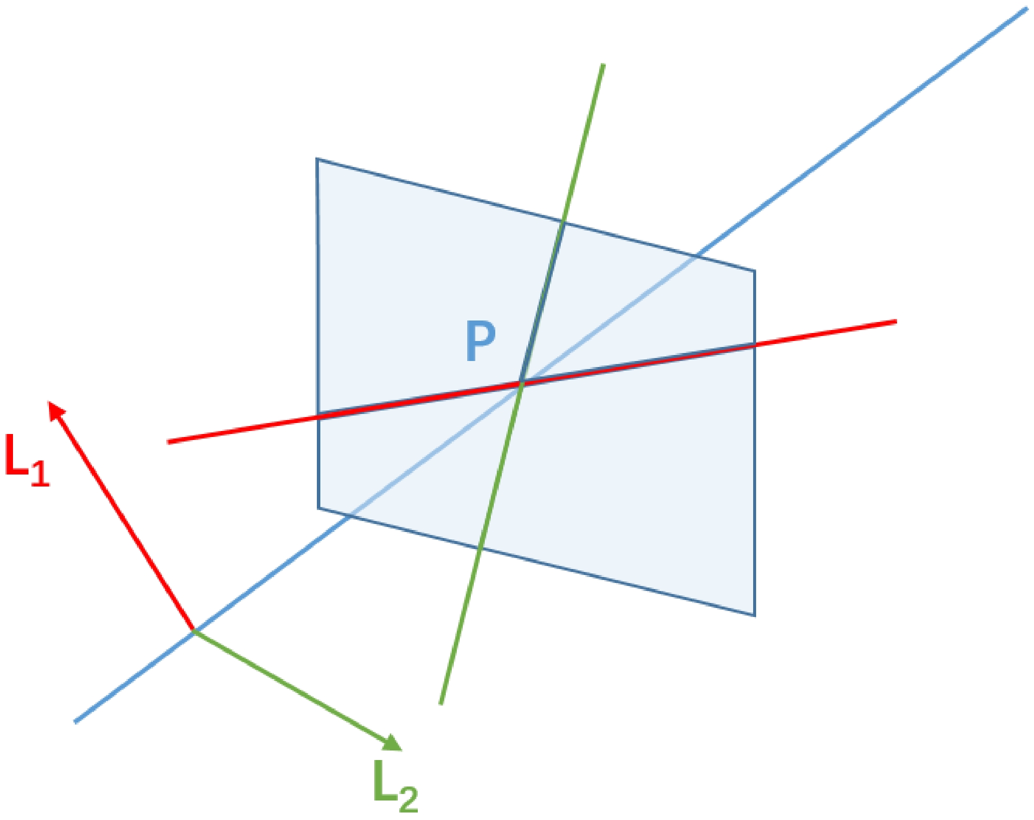
Given two points: $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$

Define a line $\boldsymbol{l} = \boldsymbol{p_1} \times \boldsymbol{p_2}$

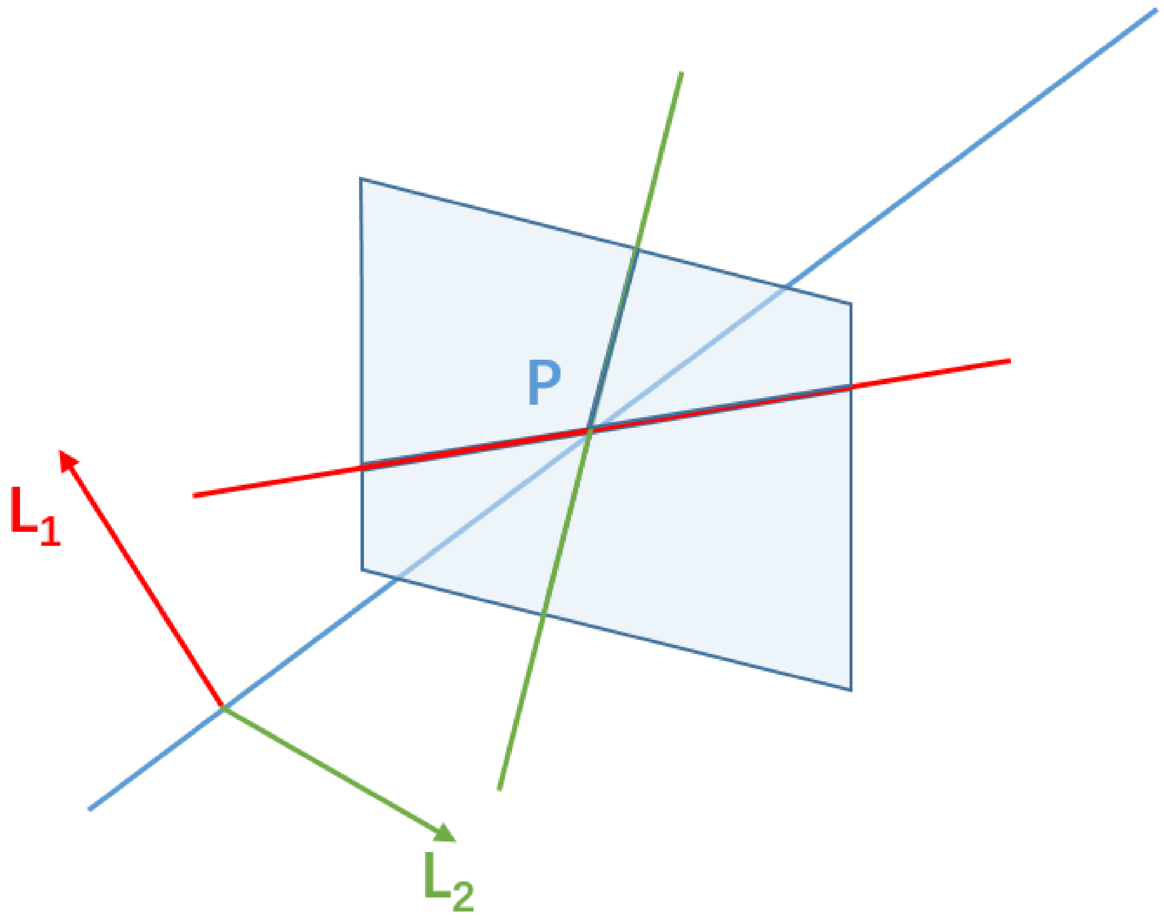
Given two lines: $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$

Define a point $\boldsymbol{p} = \boldsymbol{l_1} \times \boldsymbol{l_2}$



# Intersection point of two lines in $\mathbb{P}^2$

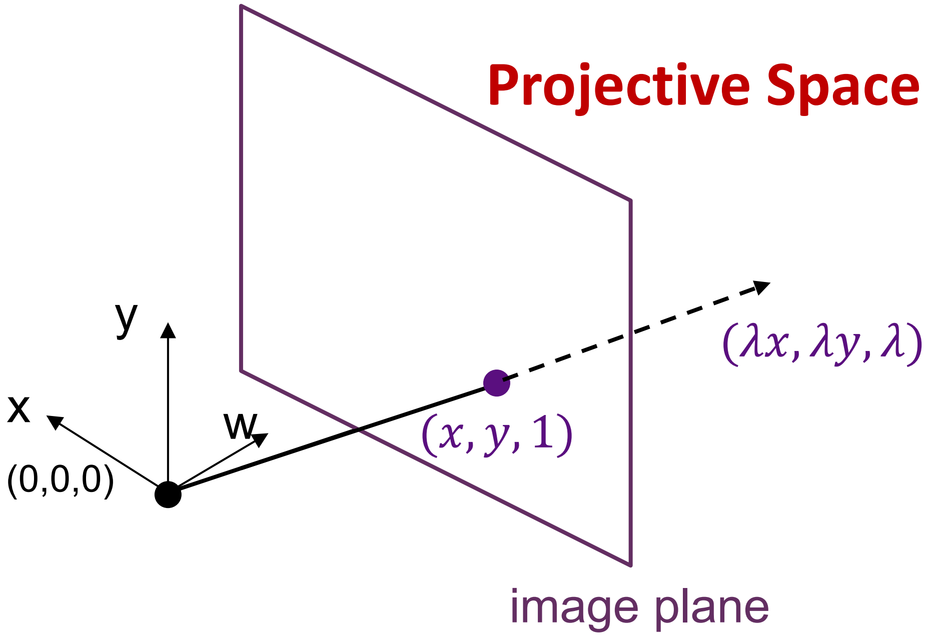


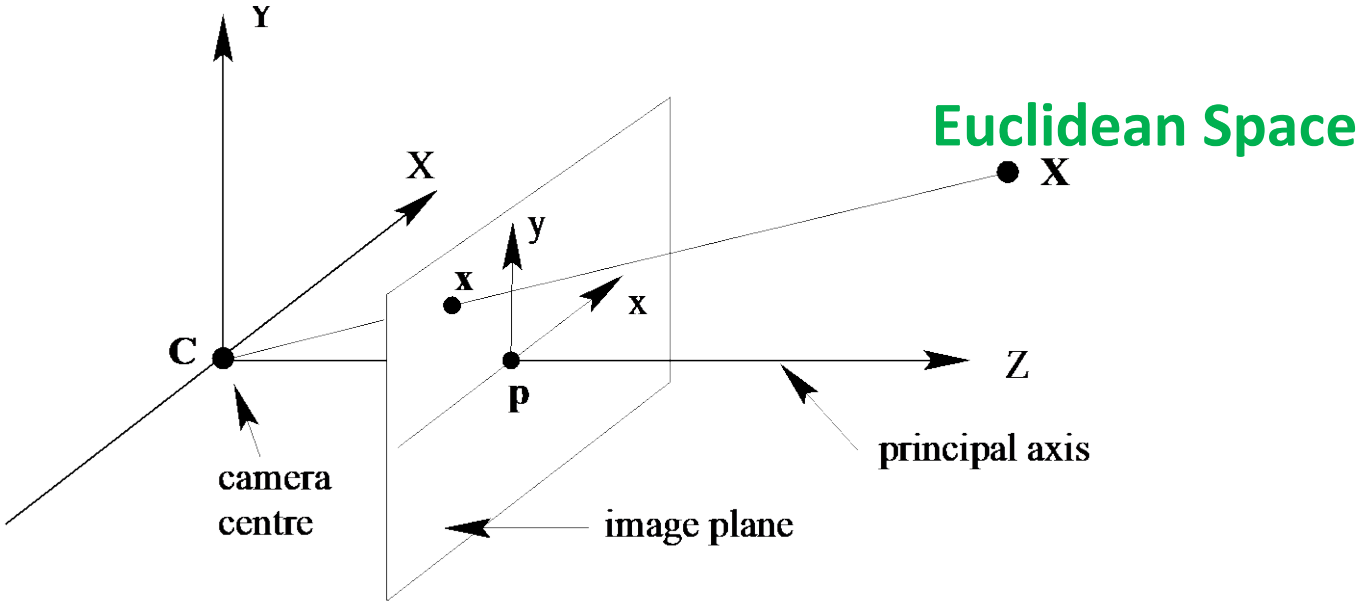

When does P have the form (x,y,0)?

# Projective geometry ↔ Euclidean interpretation

In the Euclidean interpretation, we treat $w$ as the third spatial coordinate.

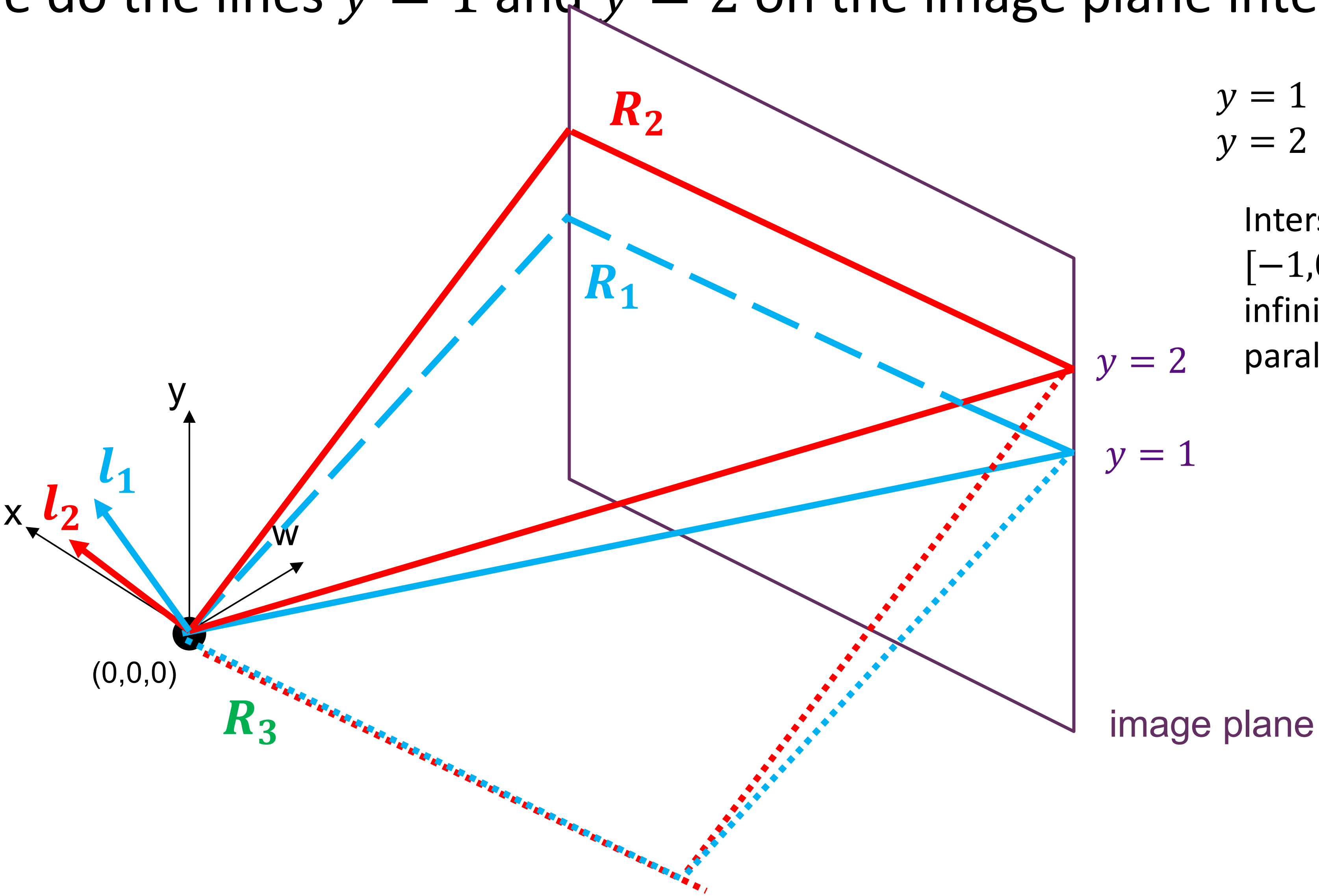
- The $w$ axis is a scaled version of the principal axis $Z$ (in camera-centric coordinates).
- The image plane is $w = 1$, same as $Z = f$
- $w = 0$ is the same as $Z = 0$. Parallel to image plane, passing through camera center.



# Exercise

- Where do the lines $y = 1$ and $y = 2$ on the image plane intersect?




$y = 1$ is $l_1 = [0,1,-1]^T$
$y = 2$ is $l_2 = [0,1,-2]^T$

Intersection = Cross product = $[-1,0,0]$ which is a point at infinity in the direction of the parallel lines!

# Extending to Any Parallel Lines

$$l = (a, b, c) \qquad\qquad l' = (a, b, c')$$

Intersection:

$$\begin{aligned} l \times l' &= l \times l' \\ &= \begin{vmatrix} i & j & k \\ a & b & c \\ a & b & c' \end{vmatrix} \\ &= (bc' - bc, ca - c'a, ab - ab)^T \\ &= (c' - c)(b, -a, 0)^T \end{aligned}$$

Any point $(x_1,x_2,0)$ is intersection of parallel lines

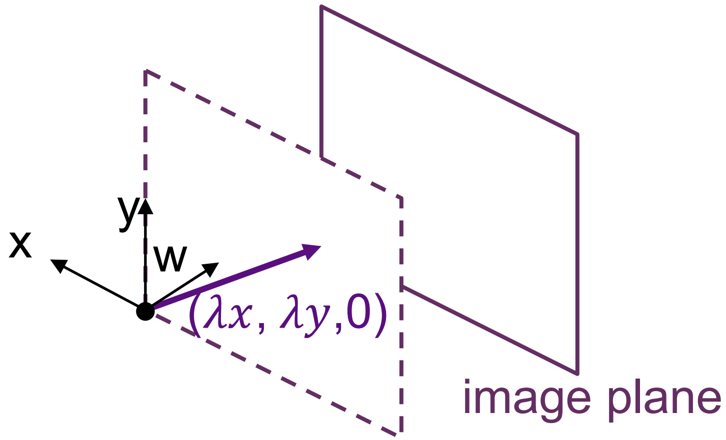
# Extending to Any Parallel Lines

- Under projective geometry,
  - All parallel lines intersect at a point at infinity
  - One point at infinity ⬄ one parallel line direction

$$\text{line } l = (a, b, c)^T \text{ intersects at } (b, -a, 0)^T$$

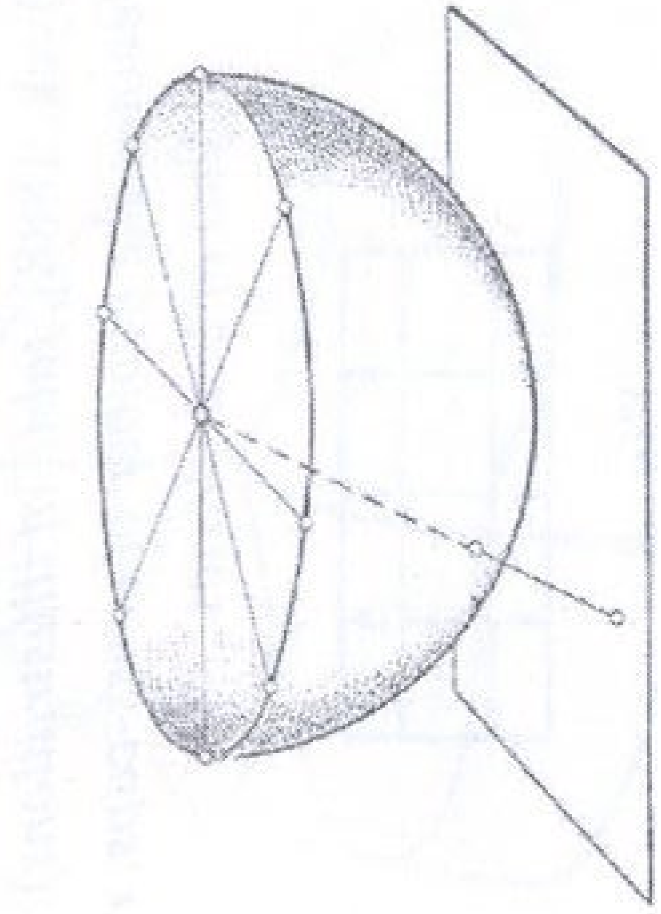
# Point at infinity / "ideal" points

- Ideal point ("point at infinity")
  - p $\cong (x, y, 0)$ – rays through camera center parallel to image plane
  - It has infinite image coordinates



# Point at infinity / “ideal” points

$$(\underbrace{x_1,x_2}_{\text{Looking-at direction}},\underbrace{0}_{\text{“Ideal” points}})$$



# "Line at infinity"

- A line passing through all ideal points i.e. point

$$l_\infty = (0, 0, 1)^T$$

- Because :

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = 0$$